

Further Maths Bridging Unit

Welcome to the fascinating and challenging subject of Further Mathematics A-level. You're clearly not satisfied with one maths A-level and would like to double the dose! Further Maths will build on much of the work in standard maths and develop it to a far more advanced level as well as building a bridge to university level study.

It is worth stating here that Further Maths is a **very challenging** course. The difficulty of the content and depth of understanding required is worlds apart from GCSE. You will be expected to have a highly driven interest in the subject as well as a hardworking attitude, and to be proactive in working with others and seeking help when needed.

Below is the bridging unit work to help best prepare you for September. The first section comprises of 10 short questions while the second section comprises of 5 longer questions. You are expected to work through these over the summer and hand in your work at the beginning of Year 12. The more effort and thought you put into this, the better prepared you will be.

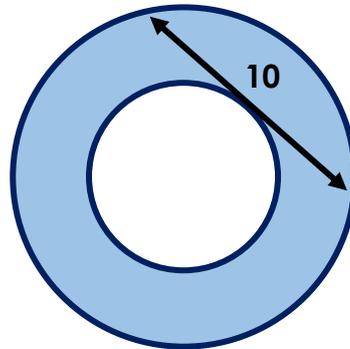
You are also encouraged to read around the subject and expand your maths knowledge beyond the curriculum. You can check out the [department reading recommendations](#) or feel free to find a book of your choice.

Have a fantastic summer and we look forward to seeing you in September!

NWS Mathematics Department

Short problems

1. Can you work out the shaded area in the diagram? (the line shown just touches the smaller circle)



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2. Find the value of

$$\frac{99}{100} \times \frac{80}{81} \times \frac{63}{64} \times \frac{48}{49} \times \frac{35}{36} \times \frac{24}{25} \times \frac{15}{16} \times \frac{8}{9} \times \frac{3}{4}$$

Write your answer in the form $\frac{a}{b}$, where a and b are positive integers with no common factors other than 1.

3. A point E lies outside the rectangle $ABCD$ such that CBE is an equilateral triangle. The area of the pentagon $ABECD$ is five times the area of the triangle CBE .

What is the ratio of the lengths $AB : AD$?

Write your answer in the form $a : 1$.

4. A sequence is defined as follows:

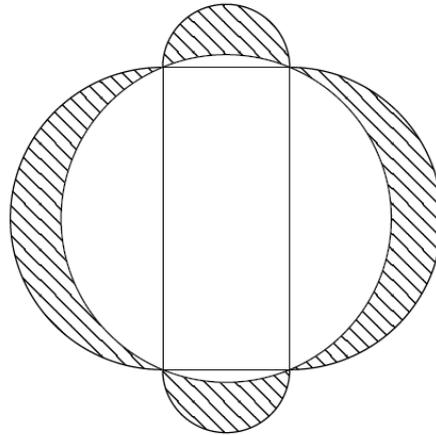
$$u_1 = 123.$$

For $n \geq 1$, define $u_{n+1} =$ the sum of the squares of the digits of u_n .

For example, $u_2 = 1^2 + 2^2 + 3^2 = 14$, $u_3 = 1^2 + 4^2 = 17$.

What is the value of u_{100} ?

5. Four semicircles are drawn on the sides of a rectangle with width 10 cm and length 24 cm. A circle is drawn that passes through the four vertices of the rectangle.



What is the value, in cm^2 , of the shaded area?

6. Alfred, Brenda, Colin, David and Erica have to sit on a row of five chairs. Alfred does not want to sit next to Brenda. David does not want to sit next to Erica.

In how many ways can these five people arrange themselves and ensure the above conditions are met?

7. Which positive integer in the range from 1 to 250 has more different prime divisors than any other integer in this range?

When $n = 5$ the product $n(n + 1)(n + 2)$ can be written as the product of four distinct primes. Indeed, when $n = 5$

$$n(n + 1)(n + 2) = 5 \times 6 \times 7 = 2 \times 3 \times 5 \times 7.$$

What is the least positive integer n such that $n(n + 1)(n + 2)$ can be written as a product of **five** distinct primes?

8. Find the value of

$$\left(\left(2^{\frac{3}{4}} + 1 \right)^2 + \left(2^{\frac{3}{4}} - 1 \right)^2 \right) \left(\left(2^{\frac{3}{4}} + 1 \right)^2 + \left(2^{\frac{3}{4}} - 1 \right)^2 - 2^2 \right).$$

9. The points $A(1,2)$ and $B(-2,1)$ are two vertices of a rectangle $ABCD$. The diagonal CA produced passes through the point $(2,9)$.

Calculate the coordinates of the vertices C and D .

10. The inequalities $x^2 + 3x + 2 > 0$ and $x^2 + x < 2$ are met by all x in the region:

a) $x < 2$; b) $-1 < x < 1$; c) $x > -1$; d) $x > -2$

11. Kelly cycles to a friend's house at an average speed of 12 km/hr. Her friend is out, so Kelly immediately returns home by the same route. At what average speed does she need to cycle home if her average speed over the whole journey is to be 15 km/hr?
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12. Evaluate the sum,

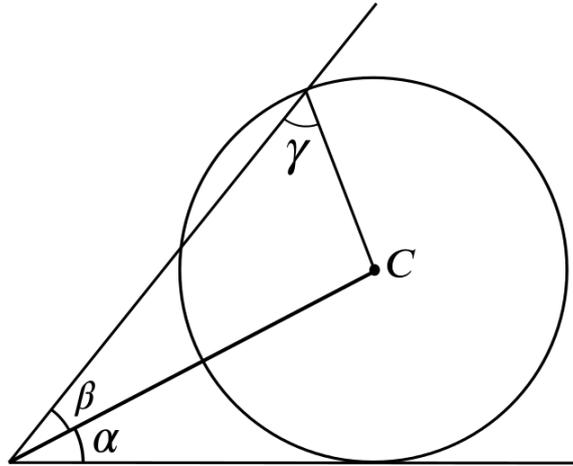
$$\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{15} + \sqrt{16}}$$

(You might want to use a calculator to get an estimate of the answer, but in order to get the exact answer you will have to do it by hand!)

13. On Monday in the village of Newton, the postman delivered either one, two, three or four letters to each house. The number of houses receiving four letters was seven times the number receiving one letter, and the number receiving two letters was five times the number receiving one letter.

What was the mean number of letters that each house received?

14. The circle in the diagram has centre C . Three angles α, β, γ are also indicated (these are Greek letters, like θ , that are often used at A-level).



Show that $\sin \beta = \sin \alpha \sin \gamma$

15. Find all real solutions of the equation,

$$(x^2 - 7x + 11)^{(x^2 - 11x + 30)} = 1$$

Yes, that is a quadratic raised to the power of another quadratic! There are more solutions than you might think.

Try plotting this graph on [desmos](https://www.desmos.com/). Why do you think part of the graph is missing?

Answers

1. 25π

2. $\frac{11}{20}$

3. $\sqrt{3} : 1$

4. 4

5. 240

6. 48

7. (a) 210

(b) 13

8. 28

9. C is $(0, -5)$ and D is $(3, -4)$

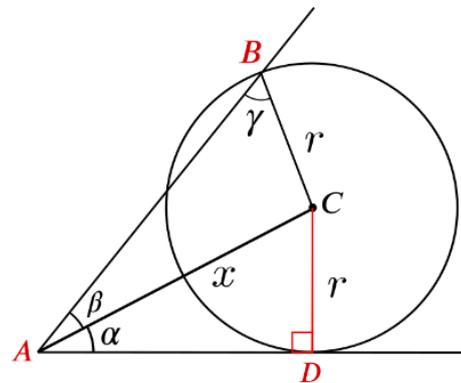
10. $-1 < x < 1$

11. 20 km/h

12. $S = 3$

13. 3

14.



$$\frac{\sin \beta}{r} = \frac{\sin \gamma}{x},$$

$$x \sin \beta = r \sin \gamma \implies x \sin \beta = x \sin \alpha \sin \gamma.$$

$$\sin \beta = \sin \alpha \sin \gamma,$$

15. $x = 2, 3, 4, 5, 6$

Longer problems

The following six problems are designed to get you to think in a bit more depth about some key mathematical concepts. Each question explores a core theme of A-level Maths that arises across many different topics. These problems will take you longer than the short ones and will require a more detailed solution. In many cases, they are quite open-ended.

You can attempt these problems in any order of your choice. No solutions are provided for these questions, however there are hints and suggestions for each one to point you in the right direction.

Graphing software is often very useful to help you understand problems and concepts. The two linked below are completely free to use and don't even require creating an account:

[Desmos Graphing Tool](#)

[Geogebra](#)

Problem 1: Straight Line Pairs

Most exercises in mathematics can be seen as construction tasks, in that we are asked to construct a mathematical object that meets certain **constraints**. Finding a locus is an example, as is solving an equation. To solve the equation $3x + 7 = 5$ is to construct a number meeting the constraint that multiplying by 3 and adding 7 results in 5. Similarly, the graph of $y = 3x + 7$ is the set of all coordinates of points (x, y) meeting the constraint that $y = 3x + 7$.

In the following problem, you are asked to construct a pair of straight lines whose x -intercepts differ by 2, whose y -intercepts differ by 3 and whose slopes differ by 1. In order to do this, it helps to work with one constraint, then add each constraint on in turn.

Sometimes you will come across constraints that are not met by any objects — for example, $x^2 = -1$ is not satisfied by any real number. Often mathematicians explore the possibility of including more objects so that an object meeting a constraint can be constructed. Thus, the whole numbers were extended to the integers (by including 0 and negatives), to the rationals, to the reals, to the complex numbers and beyond.

Problem 1: Straight Line Pairs

Construct a pair of straight lines whose x -intercepts differ by 2, y -intercepts differ by 3 and gradients differ by 1. Try starting with the case where one of the lines goes through the origin.

- ❖ How many different solutions can you find with a line through the origin as a starting point? What do you notice about the solutions?
- ❖ *You might find it helpful to try sketching your ideas on paper or use graphing software.*

Trying to solve a problem by considering a specific case such as this, is often only the first step. The next step would be to try and generalise your findings. There are many ways to approach this, below are three different cases to explore:

- ❖ We could fix the gradient of one of the lines. This is probably the simplest situation to start with.
- ❖ We could fix the y -intercept of one of the lines, e.g. $y = mx + 2$
- ❖ We could fix the x -intercept of one of the lines. We don't usually think about the x -intercepts of a straight line, so this is an interesting starting point.

Problem 2: Parabola

Averaging crops up all over the place in mathematics. We are familiar with the idea of finding an average of some data (indeed, we have more than one type of average that we can use), but there are many other contexts where what we are essentially doing is finding an average. Often, we don't explicitly describe it in that way, but being alert to the connection can be very helpful.

In statistics, you will have met various averages that represent large datasets in different ways. They are all ways of expressing a centre for the data. Similarly, we might be interested in the centres of mass of certain objects, including triangles, the midpoints of line segments, or the mean value of a function. There are other kinds of average that crop up during A level maths and further maths.

In the following question we have an example of this from coordinate geometry, which is a significant topic in A level Maths and Further Maths. The skills you have learned at GCSE will be developed further and applied to a variety of problems.

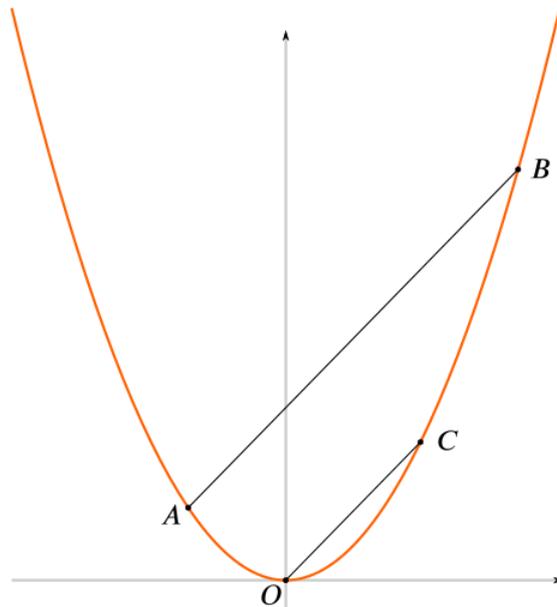
Problem 2: Parabola

Take any two points A and B on the parabola $y = x^2$.

Draw the line OC through the origin, parallel to AB , cutting the parabola again at C .

Let A have coordinates (a, a^2) , let B have coordinates (b, b^2) , and let C have coordinates (c, c^2) .

Prove that $a + b = c$.



Imagine drawing another parallel line DE , where D and E are two other points on the parabola. Extend the ideas of the previous result to prove that the midpoints of each of the three parallel lines lie on a straight line.

Problem 3: Triominoes

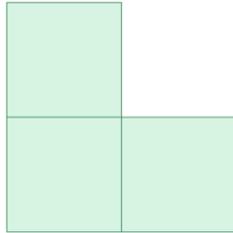
Consider the statement that 'the angles of a planar triangle add up to 180° '. Perhaps the most easily overlooked word is 'a', but it is important because it refers not to a specific triangle but to all possible triangles in the plane. The statement is saying that no matter how a triangle varies, the sum of the angles is ***invariant***.

Many if not most mathematical results can be seen as identification of some relationship or relationships that are invariant under some specified change. Invariance underlies the algebraic manipulation we use to solve equations and inequalities. For example, the value of a fraction is invariant under multiplication of the numerator and denominator by any non-zero number.

If an object such as $n^5 - n$ has a property which holds for any positive integer n , then this property remains invariant as we pass from n to $n + 1$. This is a central idea in proof by induction and offers an alternative way to prove the result mentioned above about the highest factor of $n^5 - n$ for positive integers n . When trying to prove a result by induction, we look for ways in which objects are related to smaller or previous examples, such as how terms in a sequence are related to previous terms.

Problem 3: Triominoes

A triomino is a flat L shape made from three square tiles.



A board is divided into squares the same size as the tiles. The board is 2^n by 2^n squares.

One square, anywhere on the board, is coloured blue.

We can put triominoes on the board, but they must not overlap and must not cover the blue square.

Is it possible to cover the board (apart from the blue square) with triominoes? Does it depend on where the blue square is, or on the n that determines the size of the board? Justify your answer.

- ❖ Rather than starting with the general 2^n by 2^n board, we could start with some specific values of n .
- ❖ If we colour one square of a 2 by 2 board, can we cover the rest with triominoes?
- ❖ If we colour one square of a 4 by 4 board, can we cover the rest with triominoes?
- ❖ And so on...

Problem 4: That's Odd ... or Even

Symmetry is something we all understand instinctively; it seems to be hardwired into our brains. As mathematical concepts go, symmetry is perhaps the one that people find easiest to relate to. But what exactly is symmetry and where in mathematics can we find it? If you haven't thought about this before, the answer may not be that obvious: symmetry is invariance under change.

A shape is symmetric if it remains the same when we apply a certain operation to it. In the case of shapes drawn on the plane the operations we are interested in are those that move a shape without changing distances between points. We have already met three types of such rigid motions: reflections, rotations and translations.

We usually think of symmetry as something we can see; something associated with patterns and shapes. But the more general concept of symmetry—invariance under change—means that all sorts of other objects can be symmetric too, including mathematical expressions. In the following problem you will investigate the various kinds of symmetries of different functions and their graphs, concluding with some very useful results.

Problem 4: That's Odd ... or Even

For some of the following functions, work out the values of $f(1)$, $f(-1)$, $f(2)$, $f(-2)$, and so on.

- $f(x) = x^2$
- $f(x) = 2$
- $f(x) = 1 - x^2$
- $f(x) = x^4 + 2$
- $f(x) = 1 - \frac{1}{x^2}$
- $f(x) = x^2 - 2x + 2$

What did you notice as you worked out these values? Now carefully sketch the graphs of the functions on the same set of axes.

- ❖ What did you notice as you sketched the graphs?
- ❖ Did your ideas change as you added new graphs to your axes?
- ❖ What are the points of intersection of some of the graphs?

On a new set of axes, carefully sketch graphs of the following functions. Again, for some of the functions you may find it helpful to work out $f(1)$, $f(-1)$, $f(2)$, $f(-2)$, and so on. Note down your ideas as you sketch each graph.

- $f(x) = x$
- $f(x) = \frac{1}{x}$
- $f(x) = x^3$
- $f(x) = \frac{x^3}{10}$
- $f(x) = x(x - 1)(x + 1)$
- $f(x) = 1 + \frac{1}{x}$

What did you notice this time?

Problem 5: Discriminating Quadratics

Imagine trying to balance this spoon on your finger. What would happen if your finger were near A? What if it were near B instead?



Your finger would act as a pivot and can be placed at any point on the spoon between A and B, but the behaviour of the spoon (whether it will stay balanced or the direction in which it will rotate about the pivot) depends on the position of the pivot. Imagine how the spoon would behave for each position of the pivot between A and B. At some point between A and B, the direction in which the spoon would rotate changes. This is the balancing point.

Now think about the quadratics $x^2 - 2x - 1$, $x^2 - 2x + 1$, and $x^2 - 2x + 3$. These are all of the form $x^2 - 2x + c$, but what happens to the roots as c varies continuously? In the next problem you will explore this, and how and why the discriminant (the name for the part of the quadratic formula inside the square root, $b^2 - 4ac$) determines the number of roots of a quadratic, thinking algebraically and graphically.

These examples both involve some sort of **limiting** behaviour in a system where something can vary continuously. In the first case, there is a limit point where the direction in which the spoon will topple changes. In the quadratic example, there is a value of c for which the quadratic changes from having 2 distinct real roots to 0 real roots. As c approaches this value, the two roots move closer together until they coincide.

Problem 5: Discriminating Quadratics

Below are several statements about the quadratic equation

$$ax^2 + bx + c = 0$$

where a , b and c are allowed to be any real numbers except that a is not 0.

For each statement, decide whether *MUST*, *MAY* or *CAN'T* is the correct word to use in the statement.

- ❖ To say that something *MUST* be the case, we need it to be true in all cases; we will need to give a convincing explanation (a proof) of why this must be always true.
- ❖ To show that something *CAN'T* be the case, we likewise need to give a convincing explanation (a proof) of why.
- ❖ To show that something *MAY* be the case, we need to give an example when it is true and an example when it is false. If you want a harder challenge, can you determine exactly when it is and when it is not true?

<p>① If $a < 0$, then $ax^2 + bx + c = 0$ MUST / MAY / CAN'T have real roots.</p>	<p>② If $b^2 - 4ac = 0$, then $ax^2 + bx + c = 0$ MUST / MAY / CAN'T have one repeated real root.</p>	<p>③ If $ax^2 + bx + c = 0$ has no real roots, then $ax^2 + bx - c = 0$ MUST / MAY / CAN'T have two distinct real roots.</p>
<p>④ If $\frac{b^2}{a} < 4c$, then $ax^2 + bx + c = 0$ MUST / MAY / CAN'T have two distinct real roots.</p>	<p>⑤ If $b = 0$, then $ax^2 + bx + c = 0$ MUST / MAY / CAN'T have one repeated real root.</p>	<p>⑥ The equation $ax^2 + bx + c = 0$ MUST / MAY / CAN'T have three real roots.</p>
<p>⑦ If $c = 0$, then $ax^2 + bx + c = 0$ MUST / MAY / CAN'T have real roots.</p>	<p>⑧ The equation $ax^2 + bx + c = 0$ MUST / MAY / CAN'T have the same number of real roots as $ax^2 - bx + c = 0$.</p>	<p>⑨ If $ax^2 + bx + c = 0$ has two distinct real roots, then we MUST / MAY / CAN'T have $ac < \frac{b^2}{4}$.</p>
<p>⑩ If $c > 0$, then $ax^2 + bx + c = 0$ MUST / MAY / CAN'T have two distinct real roots.</p>	<p>⑪ The equation $ax^2 + bx + c = 0$ MUST / MAY / CAN'T have the same number of real roots as $cx^2 + bx + a = 0$.</p>	<p>⑫ If $ax^2 + bx + c = 0$ has no real roots, then $-ax^2 - bx - c = 0$ MUST / MAY / CAN'T have two distinct real roots.</p>

Problem 6: Translating or Not?

Transforming things allows us to see them from many points of view. The word transformation is used to describe functions or operations that preserve some structure, and some other characteristics are changed in a structured way.

We are familiar with reflecting (which preserves the mirror line), rotating (which preserves the centre-point) and translating in 2-D and 3-D geometry; these all preserve both shape and size. Enlargements preserve shape and proportion, but not usually size, while stretches keep straight lines straight, but change shape and angles.

There are other types of geometric transformations. For instance, you may have seen a pile of coins or playing cards pushed over to make a leaning tower. These transformations preserve the base and the height, and the distance each coin is moved is a function of its height. These transformations are called shears; you will meet these in further maths.

Within mathematics, transformations are often used to move an object from a place where it is hard to work with it to a place where it is simpler. For some people moving the object is equivalent to choosing a new way to view the object. Thinking of functions as transformations of other (possibly more familiar) functions can also be useful. For example, $y = \sin x$ and $y = \cos x$ can be regarded as translations of each other, while $y = \sin^2 x$ can be thought of as a suitable transformation of $y = \cos x$.

In this last problem you will look at how a function transforms as the parameters in its definition are adjusted.

Problem 6: Translating or Not?

Sketch the graph of $y = \frac{1}{x+a}$ for different values of a . We suggest you start by trying the values $a = 0, 1, -1$.

What do you notice?

Can you explain your observations?

When sketching the graph of an unfamiliar function, it might help to think about the following:

- ❖ Where does the graph cross the axes?
- ❖ When is the function positive and when is it negative?
- ❖ Does it have any asymptotes?
- ❖ Is it increasing or decreasing?
- ❖ What happens as x gets very large?
- ❖ Is it related to any other graphs whose shape I know?

As a check, you could substitute in some values of x such as $x = 0, 1, 2, -1$.

Now sketch the graph of $y = \frac{1}{x^2+a}$ for different values of a , again starting with the values $a = 0, 1, -1$.

What do you notice this time?

Can you explain why the graphs behave in this way?

- ❖ You might find it helpful to first sketch the graph of $y = x^2 + a$ (for your chosen value of a).
- ❖ How does the graph of $y = \frac{1}{f(x)}$ relate to the graph of $y = f(x)$?

Maths Department Reading Recommendations

We are very fortunate to have a large collection of Mathematics books in our library here at Newstead, so we strongly encourage you to read around the subject as much as possible! This will not only deepen and enrich your knowledge of Mathematics but also help strengthen your university application. Here we recommend a selection of books, divided into the following categories:

Easier reads are light and fun in tone, and often quite quick to get through. However, you can still learn an awful lot of interesting things, especially about the history of the subject.

More in-depth reads are still written for a wide audience but cover sophisticated mathematical ideas to a greater level of detail. We often think of these books as the 'sweet spot' for A-level students.

Advanced reads are more formal texts, focusing on a specific area of mathematics, and written for mathematics students. They are closer in style to university textbooks/lecture notes and are more challenging but rewarding at the same time.

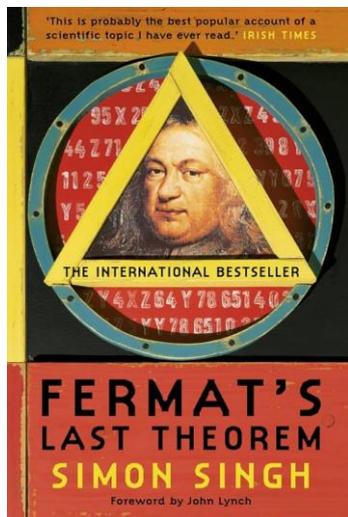
Puzzle books are great resources for fun maths problems and riddles that can be a nice break from school mathematics. You may want to buy your own copy though if you like scribbling answers and notes on the questions!

Miscellaneous books are for the two books that didn't fit into the above categories.

Enjoy your reading!

Easier reads

Fermat's Last Theorem (Simon Singh)

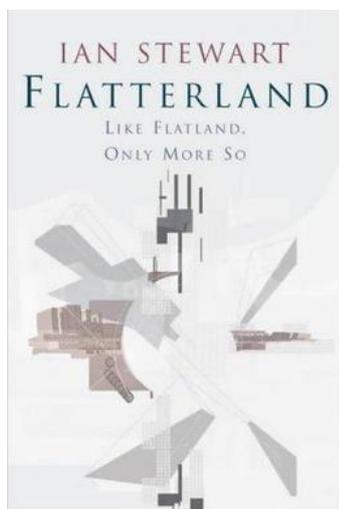


'I have a truly marvellous demonstration of this proposition which this margin is too narrow to contain.'

It was with these words, written in the 1630s, that Pierre de Fermat intrigued and infuriated the mathematics community. For over 350 years, proving Fermat's Last Theorem was the most notorious unsolved mathematical problem, a puzzle whose basics most children could grasp but whose solution eluded the greatest minds in the world. In 1993, after years of secret toil, Englishman Andrew Wiles announced to an astounded audience that he had cracked Fermat's Last Theorem. He had no idea of the nightmare that lay ahead.

In 'Fermat's Last Theorem' Simon Singh has crafted a remarkable tale of intellectual endeavour spanning three centuries, and a moving testament to the obsession, sacrifice and extraordinary determination of Andrew Wiles: one man against all the odds.

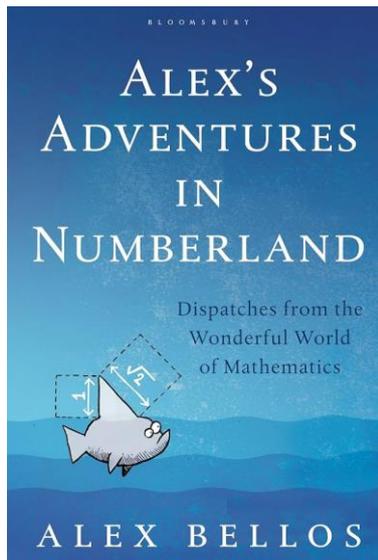
Flatterland (Ian Stewart)



Through larger-than-life characters and an inspired story line, Flatterland explores our present understanding of the shape and origins of the universe, the nature of space, time, and matter, as well as modern geometries and their applications.

The journey begins when our heroine, Victoria Line, comes upon her great-great-grandfather A. Square's diary, hidden in the attic. The writings help her to contact the Space Hopper, who tempts her away from her home and family in Flatland and becomes her guide and mentor through ten dimensions. In the tradition of Alice in Wonderland and The Phantom Toll Booth, this magnificent investigation into the nature of reality is destined to become a modern classic.

Alex's Adventures in Numberland (Alex Bellos)



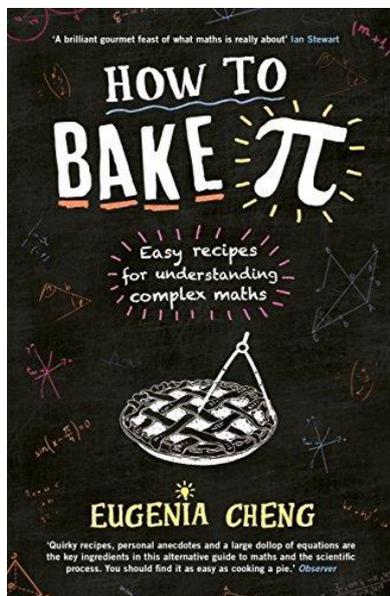
The world of maths can seem mind-boggling, irrelevant and, let's face it, boring. This groundbreaking book reclaims maths from the geeks.

Mathematical ideas underpin just about everything in our lives: from the surprising geometry of the 50p piece to how probability can help you win in any casino. In search of weird and wonderful mathematical phenomena, Alex Bellos travels across the globe and meets the world's fastest mental calculators in Germany and a startlingly numerate chimpanzee in Japan.

Packed with fascinating, eye-opening anecdotes, Alex's Adventures in Numberland is an exhilarating cocktail of history, reportage and mathematical proofs that will leave you awestruck.

More in-depth reads

How to Bake Pi (Eugenia Cheng)

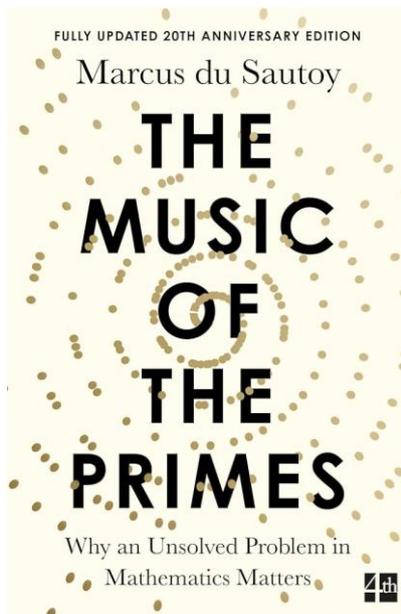


What is math? How exactly does it work? And what do three siblings trying to share a cake have to do with it?

In How to Bake Pi, math professor Eugenia Cheng provides an accessible introduction to the logic and beauty of mathematics, powered, unexpectedly, by insights from the kitchen. We learn how the bechamel in a lasagna can be a lot like the number five, and why making a good custard proves that math is easy, but life is hard. At the heart of it all is Cheng's work on category theory, a cutting-edge "mathematics of mathematics," that is about figuring out how math works.

Combined with her infectious enthusiasm for cooking and true zest for life, Cheng's perspective on math is a funny journey through a vast territory no popular book on math has explored before. So, what is math? Let's look for the answer in the kitchen.

The Music of the Primes (Marcus Du Sautoy)



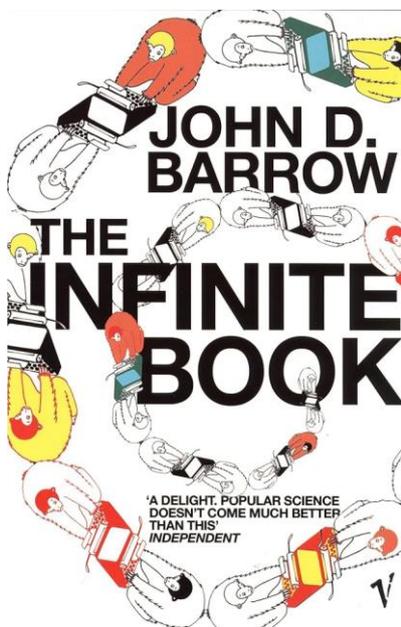
Prime numbers are the very atoms of arithmetic. They also embody one of the most tantalising enigmas in the pursuit of human knowledge. How can one predict when the next prime number will occur? Is there a formula which could generate primes? These apparently simple questions have confounded mathematicians ever since the Ancient Greeks.

In 1859, the brilliant German mathematician Bernard Riemann put forward an idea which finally seemed to reveal a magical harmony at work in the numerical landscape. Yet Riemann, a hypochondriac and a troubled perfectionist, never publicly provided a proof for his hypothesis and his housekeeper burnt all his personal papers on his death.

Whoever cracks Riemann's hypothesis will go down in history, for it has implications far beyond mathematics.

In science, it has critical ramifications in Quantum Mechanics, Chaos Theory, and the future of computing. Pioneers in each of these fields are racing to crack the code and a prize of \$1 million has been offered to the winner. As yet, it remains unsolved.

The Infinite Book (John D. Barrow)



Everything you might want to know about infinity - in history and all the way to today's cutting-edge science.

Infinity is surely the strangest idea that humans have ever had. Where did it come from and what is it telling us about our Universe? Can there actually be infinities? Can you do an infinite number of things in a finite amount of time? Is the Universe infinite?

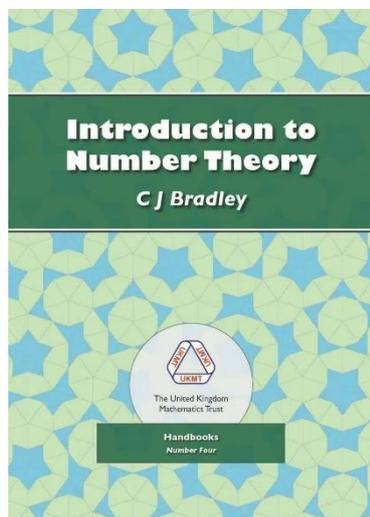
Infinity is also the place where things happen that don't. What is it like to live in a Universe where nothing is original, where you can live forever, where anything that can be done, is done, over and over again?

These are some of the deep questions that the idea of the infinite pushes us to ask. Throughout history, the infinite has been a dangerous concept. Many have lost their lives, their careers, or their freedom for talking about it.

The Infinite Book will take you on a tour of these dangerous questions and the strange answers that scientists, mathematicians, philosophers and theologians have come up with to deal with its threats to our sanity.

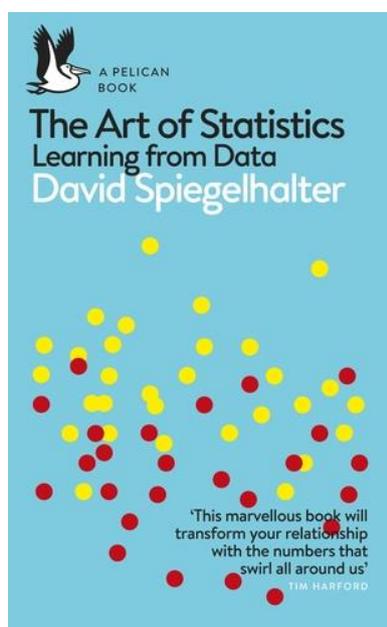
Advanced reads

Introduction to Number Theory (C J Bradley)



By C. J. Bradley The aim of this 200 page book is to enable talented students to tackle the sort of problems on number theory which are set in mathematics competitions. Topics include primes and divisibility, congruence arithmetic and the representation of real numbers by decimals. A useful summary of techniques and hints is included. This is a fully revised and extended edition of a book which was initially published as part of the composite volume 'Introductions to Number Theory and Inequalities'. The author was an Oxford University lecturer and teacher at Clifton College, Bristol.

The Art of Statistics (David Spiegelhalter)



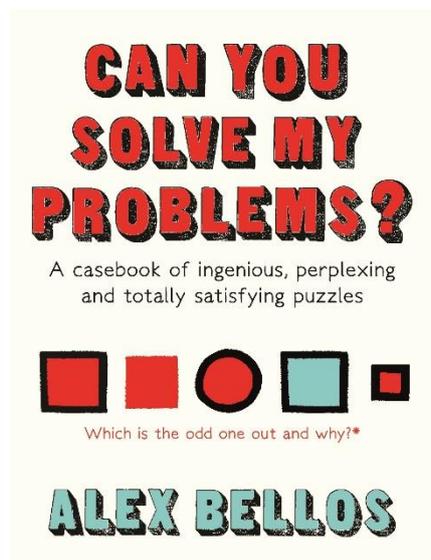
Statistics has played a leading role in our scientific understanding of the world for centuries, yet we are all familiar with the way statistical claims can be sensationalised, particularly in the media. In the age of big data, as data science becomes established as a discipline, a basic grasp of statistical literacy is more important than ever.

In *The Art of Statistics*, David Spiegelhalter guides the reader through the essential principles we need in order to derive knowledge from data. Drawing on real world problems to introduce conceptual issues, he shows us how statistics can help us determine the luckiest passenger on the Titanic, whether serial killer Harold Shipman could have been caught earlier, and if screening for ovarian cancer is beneficial.

How many trees are there on the planet? Do busier hospitals have higher survival rates? Why do old men have big ears? Spiegelhalter reveals the answers to these and many other questions - questions that can only be addressed using statistical science.

Puzzle books

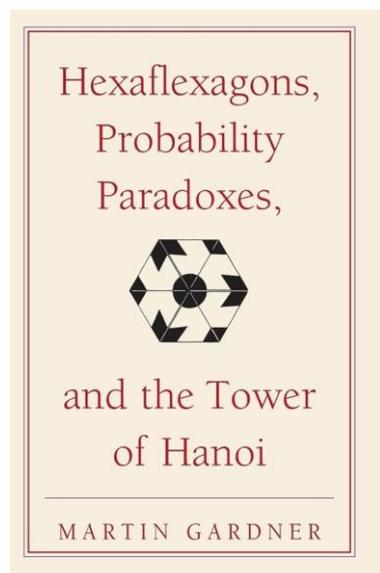
Can You Solve My Problems? (Alex Bellos)



Are you smarter than a Singaporean ten-year-old? Can you beat Sherlock Holmes? If you think the answer is yes – I challenge you to solve my problems. Here are 125 of the world's best brainteasers from the last two millennia, taking us from ancient China to medieval Europe, Victorian England to modern-day Japan, with stories of espionage, mathematical breakthroughs and puzzling rivalries along the way.

Pit your wits against logic puzzles and kinship riddles, pangrams and river-crossing conundrums. Some solutions rely on a touch of cunning, others call for creativity, others need mercilessly logical thought. Some can only be solved by 2 per cent of the population. All are guaranteed to sharpen your mind. Let's get puzzling!

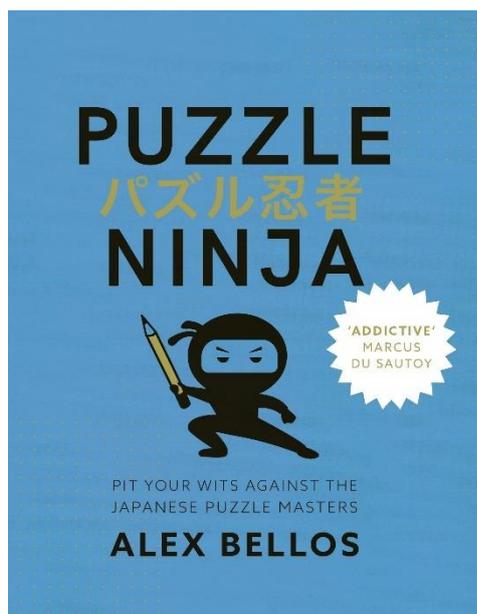
Hexaflexagons, Probability, Paradoxes, and the Tower of Hanoi (Martin Gardner)



Paradoxes and paper-folding, Moebius variations and mnemonics, fallacies, magic squares, topological curiosities, parlor tricks, and games ancient and modern, from Polyominoes, Nim, Hex, and the Tower of Hanoi to four-dimensional ticktacktoe. These mathematical recreations, clearly and cleverly presented by Martin Gardner, delight and perplex while demonstrating principles of logic, probability, geometry, and other fields of mathematics.

This book of the earliest of Gardner's enormously popular Scientific American columns and puzzles continues to challenge and fascinate readers. Now the author, in consultation with experts, has added updates to all the chapters, including new game variations, mathematical proofs, and other developments and discoveries.

Puzzle Ninja (Alex Bellos)

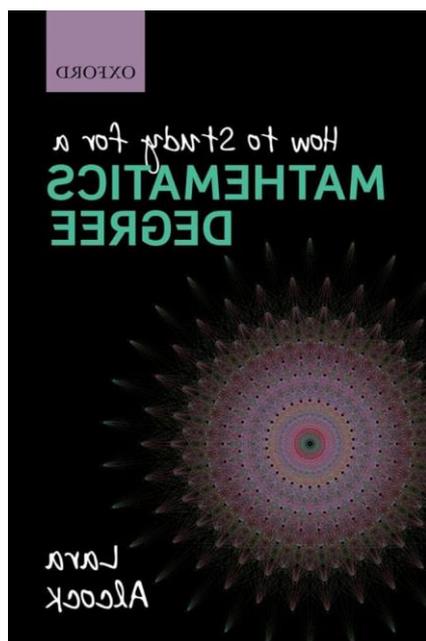


Puzzles are so enjoyable. They get your brain sparking and the competitive spirit flowing. Solving them is one of life's simple pleasures. The puzzle masters of Japan create the world's most satisfying puzzles, so Alex Bellos travelled to Tokyo to meet them. These enigmatologists include the god-father of Sudoku, the winner of the WorldPuzzle Championships, an inspiring teacher who uses games to enliven his students' maths lessons, and the puzzle poet whose name has become a Sudoku-solving technique. They use noms de guerre — Edamame, Lenin, Teatime, Sesame Egg — and each has a distinctive style. What unites them are their megawatt brains and the beauty of their hand-crafted puzzles, which will challenge and sharpen your mind.

Bellos has collected over 200 of their most ingenious puzzles, rated easy to excruciating, and introduces over 20 new types of addictive problems including Shakashaka and Marupeke. Arm yourself with pencil, eraser and laser-like focus. Let's get puzzling . . .

Miscellaneous books

How to Study for a Mathematics Degree (Lara Alcock)

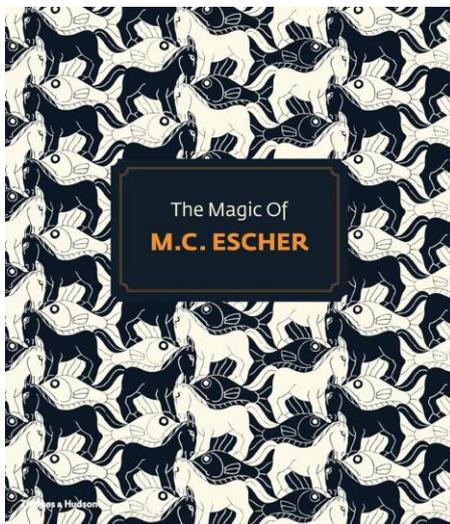


Every year, thousands of students go to university to study mathematics (single honours or combined with another subject). Many of these students are extremely intelligent and hardworking, but even the best will, at some point, struggle with the demands of making the transition to advanced mathematics. Some have difficulty adjusting to independent study and to learning from lectures. Other struggles, however, are more fundamental: the mathematics shifts in focus from calculation to proof, so students are expected to interact with it in different ways. These changes need not be mysterious - mathematics education research has revealed many insights into the adjustments that are necessary - but they are not obvious and they do need explaining.

This no-nonsense book translates these researchbased insights into practical advice for a student audience. It covers every aspect of studying for a mathematics degree, from the most abstract intellectual challenges to the everyday business of interacting with lecturers and making good use of study time.

Part 1 provides an in-depth discussion of advanced mathematical thinking, and explains how a student will need to adapt and extend their existing skills in order to develop a good understanding of undergraduate mathematics. Part 2 covers study skills as these relate to the demands of a mathematics degree. It suggests practical approaches to learning from lectures and to studying for examinations while also allowing time for a fulfilling all-round university experience. The first subject-specific guide for students, this friendly, practical text will be essential reading for anyone studying mathematics at university.

The Magic of M.C. Escher (M.C. Escher)



M.C. Escher's mesmerizing artworks create a realm of enchantment and illusion, and tens of thousands of people everywhere have fallen under his spell. This exciting new book deepens our understanding of this artist, who has been the subject of some of the most successful books Abrams has published over the past half century.

Brilliantly interweaving well-known prints with numerous unpublished drawings, incredible details, the artist's eloquent words, and observations by Escher expert J.L. Locher, this fresh presentation -- which includes 10 dynamic full-colour gatefolds -- reveals Escher's tireless quest for new visual

concepts of space and time. Here at last is a book that does justice to Escher's invention, which is, if anything, increasingly relevant in today's sophisticated world of 3-D computer graphics.